Kinetic theory for radiation interacting with sound waves in ultrarelativistic pair plasmas

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A kinetic theory for radiation interacting with sound waves in an ultrarelativistic electron–positron plasma is developed. It is shown that the effect of a spatial spectral broadening of the electromagnetic pulse is to introduce a reduction of the growth rates for the decay and modulational instabilities. Such spectral broadening could be due to a finite pulse coherence length, or through the use of random phase filters, and would stabilize the propagation of electromagnetic pulses.

The physics of electron–positron plasmas is important for understanding the pulsar environments [1, 2] and laboratory plasmas irradiated by intense lasers [3, 4], where quantum field effects come into play [5]. Pair plasmas are also believed to be important in the early universe, in supernova remnants and active galactic nuclei, and in gamma-ray bursts [6, 7]. A pair plasma can be created in these environments by collisions between strongly accelerated particles [8]. There is also a possibility of pair creation via high-energy curvature radiation photons in the vicinity of strongly magnetized astrophysical objects, triggered by charged particles streaming along the curved magnetic field [9]. This results in large quantities of positrons produced close to pulsar polar caps [10, 11]. In laboratory environments, laser experiments with focal spot intensities exceeding $10^{20}\,\mathrm{W/cm^2}$ have demonstrated the production of MeV electrons and evidence of positrons via laser multiphoton–gamma photon interactions [12, 13] as well as via electron collisions [14, 15].

When the intensity of the radiation is increased, the pair plasma particles attain rela-

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tivistic quiver velocities, and the plasma dynamics turns out to be essentially nonlinear, see e.g. Refs. [16, 17, 18, 19]. New decay instabilities, which are due to relativistic effects, can thus appear [20]. The nonlinear propagation of radiation in electron–positron plasmas has therefore been suggested to be the reason behind the high effective temperatures of pulsar radio emissions [21] as well as a possible source of the large frequency shifts in such emissions [22].

In the present paper, we will study the effects of partial coherence on the nonlinear radiation propagation in pair plasmas. It is shown that a spatial spectral broadening of the electromagnetic pulse can lead to a reduced growth rate for the decay and modulational instabilities. This type of pulse broadening could be due to a finite pulse coherence length (as in ultra-short laser pulse application), or through the use of random phase filters. The effect of the broadening is to stabilize the propagation of electromagnetic pulses in pair plasmas.

The interaction between electromagnetic radiation and sound waves in pair plasmas with ultrarelativistic temperatures is given by the system of equations [23]

$$\left(\partial_t^2 - c^2\nabla^2 + 2\omega_p^2\right)\boldsymbol{A} + \omega_p^2\left(\frac{4}{3}\frac{n_1}{n_0} - \frac{e^2}{m^2(T)c^4}|\boldsymbol{A}|^2\right)\boldsymbol{A} = 0, \tag{1}$$

and

$$\left(\partial_t^2 - u^2\nabla^2\right)\frac{n_1}{n_0} = \frac{e^2c^2}{32T^2}\nabla^2|\boldsymbol{A}|^2, \tag{2}$$

where $\boldsymbol{A}$ is the vector potential of the radiation, $\omega_p = (\pi n_0 e^2 c^2/T)^{1/2}$ is the effective plasma frequency, $m(T) = 4T/c^2$ is the effective mass, T is the electron and positron temperature, n_0 is the unperturbed plasma density, n_1 $(\ll n_0)$ is the density perturbation, c is the speed of light in vacuum, e is the magnitude of the electron charge, $u = [(c^2/3)(1 + \sigma T^3/4n_0)]^{1/2}$ is the sound speed in a relativistically hot pair plasma, $\sigma = 4\pi^2/45\hbar^3c^3$, and $\hbar$ is the Planck constant divided by 2π. A similar system has been previously derived for a cold plasma [24], and recently for a warm dust-laden plasma [25].

A kinetic description of the radiation propagation may be obtained by the introduction of the Wigner density matrix (see e.g. [26, 27, 28, 29]

$$\rho_{ij}(\boldsymbol{r}, t, \boldsymbol{k}, \omega) = \frac{1}{(2\pi)^4}\int d\tau\, d\boldsymbol{\zeta}\, \exp(i\boldsymbol{k}\cdot\boldsymbol{\zeta} - i\omega\tau)\langle A_i^*(\boldsymbol{r}+\boldsymbol{\zeta}/2, t+\tau/2)A_j(\boldsymbol{r}-\boldsymbol{\zeta}/2, t-\tau/2)\rangle, \tag{3}$$

where the angular brackets denote the ensemble average [30]. The Wigner density matrix generalizes the the classical distribution function, and satisfies

$$I \equiv \langle |\boldsymbol{A}|^2 \rangle = \int d\omega \, d\boldsymbol{k} \, \rho \tag{4}$$

where $\rho = \mathrm{Tr}(\rho_{ij})$ is the trace of the state density matrix. We note that for a monochromatic pump wave $\boldsymbol{A} = \boldsymbol{A}_0 \exp(i\boldsymbol{k}_0 \cdot \boldsymbol{r} - i\omega_0 t) + \text{c.c.}$, we have $I = 2|\boldsymbol{A}_0|^2$. As there is no polarization mixing and the ponderomotive force term only depends on I, it will here suffice to use ρ as our radiation field description. Applying the time derivative to the definition (3), using Eq. (1), and taking the trace of the resulting equation we obtain

$$\omega \partial_t \rho + c^2 \boldsymbol{k} \cdot \boldsymbol{\nabla} \rho + \omega_{\mathrm{p}}^2 \left(\frac{4}{3} \frac{n_1}{n_0} - \frac{e^2}{m^2(T)c^4} I \right) \sin \left[\frac{1}{2} \left(\overleftarrow{\partial_t} \overrightarrow{\partial_\omega} - \boldsymbol{\nabla} \cdot \boldsymbol{\nabla}_k \right) \right] \rho = 0 \tag{5}$$

noting that the term proportional to the squared plasma frequency in Eq. (1) only contributes with a constant phase factor and therefore does not appear in Eq. (5). The equations (2), (4), and (5) form a closed system describing the nonlinear interaction of sound waves and partially coherent radiation in pair plasmas with ultrarelativistic temperatures.

Next, we analyze the modulational instability [31, 32] of the coupled system of equations (2), (4), and (5), by letting $n_1 = \tilde{n}_1 \exp(i\boldsymbol{K} \cdot \boldsymbol{r} - i\Omega t)$ and $\rho = \rho_0(\boldsymbol{k}, \omega) + \tilde{\rho} \exp(i\boldsymbol{K} \cdot \boldsymbol{r} - i\Omega t)$, where $|\tilde{\rho}_1| \ll \rho_0$ and $\boldsymbol{K}$ and Ω are the perturbation wave vector and frequency, respectively. We then linearize with respect to the perturbations to obtain the nonlinear dispersion relation

$$\Omega^2 - K^2 u^2 = -\frac{\omega_{\mathrm{p}}^2 e^2}{48 T^2} \left[K^2 c^2 - \frac{3}{2}(\Omega^2 - K^2 u^2) \right] \int d\omega \, d\boldsymbol{k} \, \frac{\rho_{0+} - \rho_{0-}}{\Omega \omega - \boldsymbol{K} \cdot \boldsymbol{k} c^2}, \tag{6}$$

where $\rho_{0\pm} \equiv \rho_0(\boldsymbol{k} \pm \boldsymbol{K}/2, \omega \pm \Omega/2)$. For a monochromatic background distribution function $\rho_0(\boldsymbol{k}, \omega) = I_0 \delta(\boldsymbol{k} - \boldsymbol{k}_0) \delta(\omega - \omega_0)$, the expression (6) reduces to [23]

$$\Omega^2 - K^2 u^2 = -\frac{I_0 \omega_{\mathrm{p}}^2 e^2}{24 T^2} \left[K^2 c^2 - \frac{3}{2}(\Omega^2 - K^2 u^2) \right] \left(\frac{1}{D_-} + \frac{1}{D_+} \right), \tag{7}$$

where

$$D_\pm \equiv K^2 c^2 - \Omega^2 \mp 2\omega_0(\Omega - \boldsymbol{K} \cdot \boldsymbol{v}_g), \tag{8}$$

and $v_g = k_0 c^2/\omega_0$. We note that (7) agrees with (4) in Ref. [23] for decay processes when $D_- \approx 0$.

Next, we consider the case of partial coherence. We will here restrict our consideration to the one-dimensional case. Assuming that the background electromagnetic wave has a random phase $\varphi(z)$ such that $\langle \exp[-i\varphi(z+\zeta/2)] \exp[i\varphi(z-\zeta/2)] \rangle = \exp(-\Delta|\zeta|)$, the quasi-particle distribution takes the Breit–Wigner form [28, 33]

$$\rho_0(k, \omega) = \frac{I_0}{\pi} \frac{\Delta}{(k-k_0)^2 + \Delta^2} \delta(\omega - \omega_0). \tag{9}$$

Here Δ is the half-width of the distribution centered around k_0. Then the dispersion relation (6) can be written as

$$\Omega^2 - K^2 u^2 = -\frac{I_0 \omega_p^2 e^2}{24 T^2} \left[K^2 c^2 - \frac{3}{2}(\Omega^2 - K^2 u^2) \right] \left(\frac{1}{D_- + 2iK\Delta c^2} + \frac{1}{D_+ - 2iK\Delta c^2} \right). \tag{10}$$

We next normalize to dimensionless variables according to $\omega_0 \rightarrow \omega_0/\omega_p$, $\Omega \rightarrow \Omega/\omega_p$, $K \rightarrow Kc/\omega_p$, $\Delta \rightarrow \Delta c/\omega_p$, $u \rightarrow u/c$, $v_g \rightarrow v_g/c$, and $I_0 \rightarrow I_0 e^2/24T^2$. In the case of the decay instability, we neglect the self-modulation of the electromagnetic wave, and we also have $D_- \approx 0$. Neglecting the Ω^2 term in D_-, we have plotted the growth rate Γ of the decay instability in Fig. 1. In Fig. 2 we have plotted the growth rate for the modulational instability. In both Figs. 1 and 2 the damping effect of the spatial spectral broadening can be seen. We have chosen the normalized parameter values $\omega_0 = 1$, $I_0 = 0.1$, $v_g = 0.1$, and $u = 0.2$ in both Figs. 1 and 2.

We also plot the case of an over-dense pair plasma, such that $\omega_0 = 0.2$ in Fig. 3. The other parameters remain the same as above.

In Fig. 4 we display the modulational instability growth rate for an under-dense pair plasma, such that $\omega_0 = 2$. The remaining parameters remain the same as above.

To summarize, we have consider the nonlinear propagation of spatial broadband electromagnetic waves in a relativistically hot electron-positron plasma. For this purpose, we have introduced the Wigner transformation on a pair of equations comprising the nonlinear Schrödinger equation and a radiation pressure driven sound equation. As a result, we obtain a wave kinetic equation that is appropriate to derive a nonlinear dispersion relation. The latter exhibits that the growth rates of the decay and modulational instabilities are reduced when electromagnetic waves a broad spatial bandwidth. Hence, the electromagnetic waves
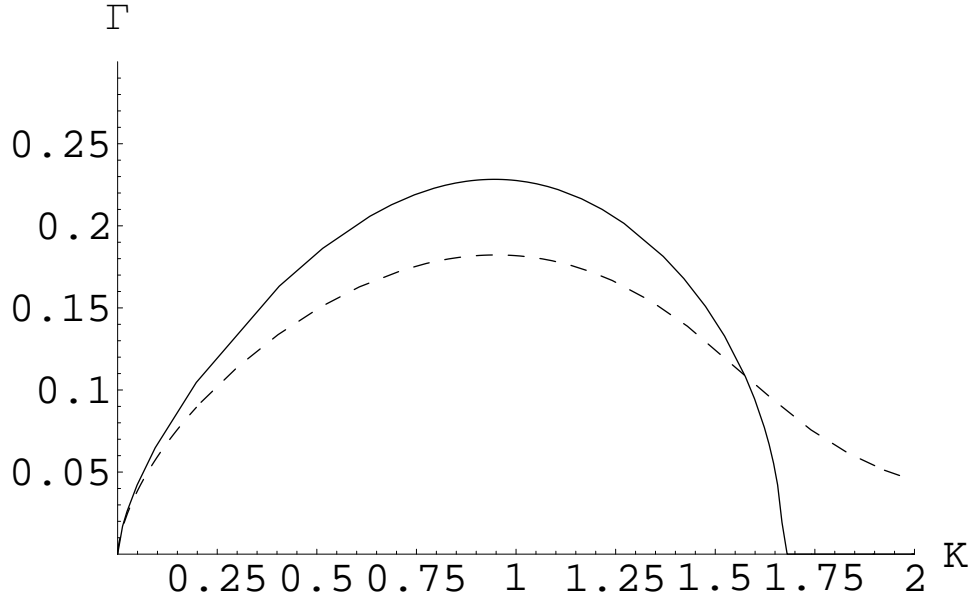
FIG. 1: The decay growth rate plotted as a function of K. The full curve has $\Delta = 0$, while the dashed curve has $\Delta = 0.25$.

would be able to propagate over long distances without losing their energy. Our results should be useful in understanding the nonlinear propagation of partially coherent electromagnetic waves in the early universe as well as in forthcoming laser-plasma interaction experiments.

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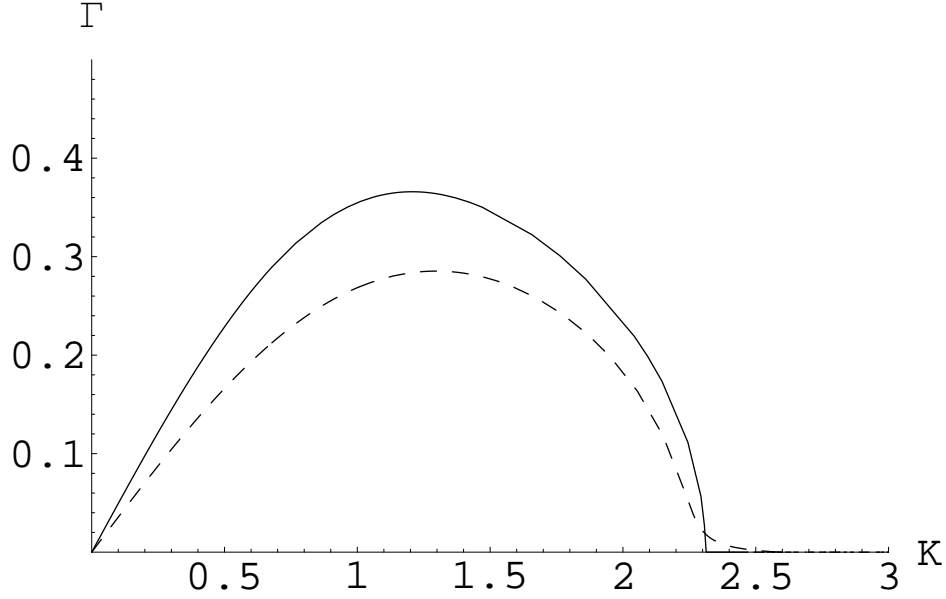
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FIG. 2: The modulational instability growth rate plotted as a function of K. The full curve has $\Delta = 0$, while the dashed curve has $\Delta = 0.25$.

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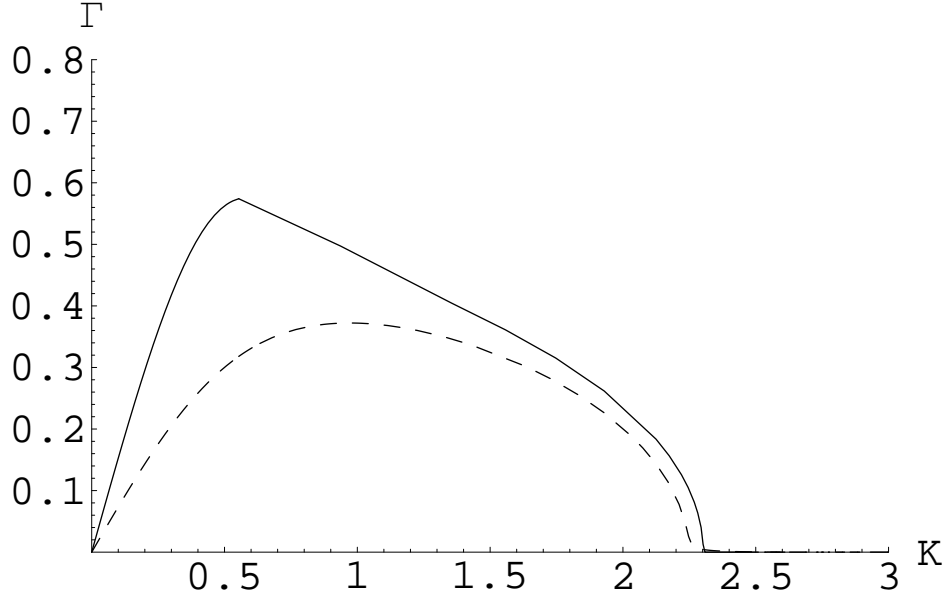[25] N. L. Tsintsadze, Z. Ehsan, H. A. Shah, and G. Murtaza, Phys. Plasmas **13**, 072103 (2006).

FIG. 3: The modulational instability growth rate plotted as a function of K in the case of an over-dense pair plasma ($\omega_0 = 0.2$). The full curve has $\Delta = 0$, while the dashed curve has $\Delta = 0.25$.

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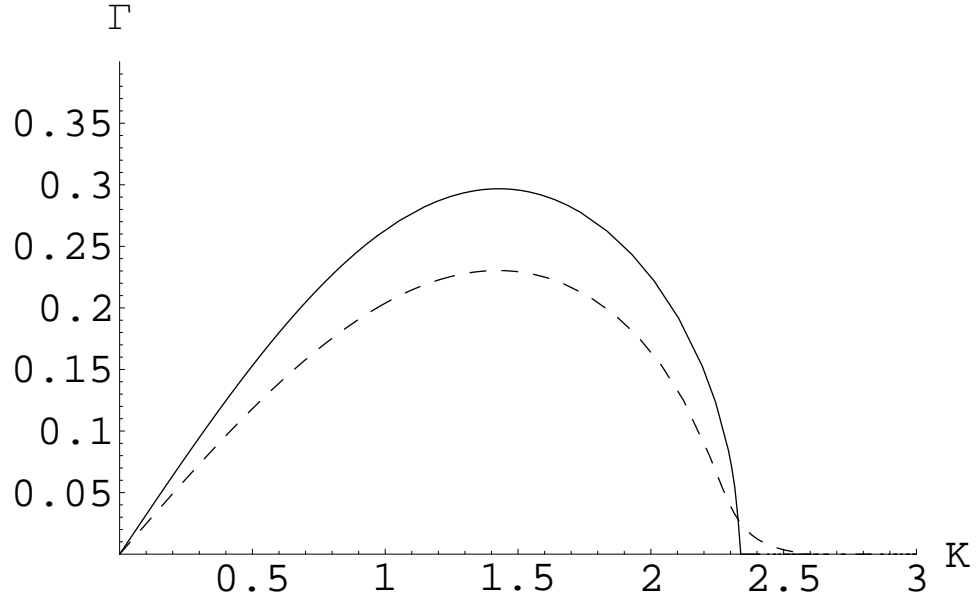[33] G. Breit and E. Wigner, Phys. Rev. **49**, 519 (1936).

FIG. 4: The modulational instability growth rate plotted as a function of K in the case of an under-dense pair plasma ($\omega_0 = 2$). The full curve has $\Delta = 0$, while the dashed curve has $\Delta = 0.25$.